International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

CHARACTERIZATION OF INTUITIONISTIC MULTI-FUZZY NORMAL SUBGROUP

S.Balamurugan
Department of Mathematics,
Velammal College of Engineering & Technology, Madurai-625 009,
Tamilnadu, India.

ABSTRACT For any intuitionistic multi-fuzzy set $A = \{ < x , \mu_A(x) , \nu_A(x) > : x \in X\}$ of an universe set X, we study the crisp multi-set $\{ x \in X : \mu_i(x) \geq \alpha_i , \nu_i(x) \leq \beta_i , \forall i \}$ of X . In this paper, an attempt has been made to study some algebraic nature of intuitionistic multi-fuzzy normal subgroup and their properties are discussed.

Keywords Intuitionistic fuzzy set (IFS), Intuitionistic multi-fuzzy set (IMFS), Intuitionistic multi-fuzzy subgroup (IMFSG), Intuitionistic multi-fuzzy normal subgroup (IMFNSG).

Mathematics Subject Classification 20N25, 03E72, 08A72, 03F55, 06F35, 03G25, 08A05

1. INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh [14] several researches were conducted on the generalization of the notion of fuzzy set. The idea of intuitionistic fuzzy set was given by Krassimir.T.Atanassov [1]. An intuitionistic fuzzy set is characterized by two functions expressing the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of elements of the universe to the IFS. Among the various notions of higher-order fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. An element of a multi-fuzzy set can occur more than once with possibly the same or different membership values. In 2011, P.K.Sharma [12] initiated the concept of Intuitionistic fuzzy groups. T.K.Shinoj and Sunil Jacob John [13] was introduced the concept of Intuitionistic multi-fuzzy set in the year of 2013.

R.Muthuraj and S.Balamurugan [8] introduced the new algebraic structure Intuitionistic multi-fuzzy subgroup in 2014. In this paper we study intuitionistic multi-fuzzy normal subgroup and its properties. This paper is an attempt to combine the two concepts: intuitionistic multi-fuzzy sets and multi-fuzzy subgroups together by introducing a new concept called intuitionistic multi-fuzzy normal subgroups.

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition [14]

Let X be a non-empty set. Then a **fuzzy set** $\mu : X \to [0,1]$.

2.2 Definition [7, 10, 11]

Let X be a non-empty set. A **multi-fuzzy set** A of X is defined as $A = \{ < x, \mu_A(x) > : x \in X \}$ where $\mu_A = (\mu_1, \mu_2, \dots, \mu_k)$, that is, $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$ and $\mu_i : X \to [0,1]$, $\forall i=1,2,\dots,k$. Here k is the finite dimension of A. Also note that, for all i, $\mu_i(x)$is a decreasingly ordered sequence of elements. That is, $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_k(x)$, $\forall x \in X$.

2.3 Definition [1]

Let X be a non-empty set. An **Intuitionistic Fuzzy Set (IFS)** A of X is an object of the form $A = \{< x, \mu(x), \nu(x) > : x \in X\}$, where $\mu : X \to [0, 1]$ and $\nu : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively with $0 \leq \mu(x) + \nu(x) \leq 1$, $\forall x \in X$.

2.4 Remark [1]

(i) Every fuzzy set A on a non-empty set X is obviously an intuitionistic fuzzy set having the form $A = \{< x, \mu(x), 1\text{-}\mu(x) > : x \in X\}$.

(ii) In the definition 2.3, When $\mu(x)+\nu(x) = 1$, that is, when $\nu(x) = 1-\mu(x) = \mu^c(x)$, A is called fuzzy set.

2.5 Definition [8, 13]

Let $A=\{<x, \mu_A(x), \nu_A(x)>: x \in G\}$, where $\mu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \dots \mu_{A_k}(x))$ and $\nu_A(x) = (\nu_{A_1}(x), \nu_{A_2}(x), \nu_{A_3}(x) \dots \nu_{A_k}(x))$ such that $0 \leq \mu_{A_1}(x) + \nu_{A_1}(x) \leq 1$, $\forall x \in G$,

$\mu_{A_1}: G \to [0,1]$ and $\nu_{A_1}: G \to [0,1]$ for all i = 1, 2, …, k. Here, $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq \mu_{A_3}(x) \geq \ldots \geq \mu_{A_k}(x)$, for all $x \in G$. That is, μ_{A_1}'s are decreasingly ordered sequence. Then the set A is said to be an **intuitionistic multi-fuzzy set (IMFS)** with dimension k of G.

2.6 Remark

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.7 Definition [8, 13]

Let $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > : x \in X \}$ be any two IMFS's having the same dimension k of X. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.

(iii) $A^C = \{ < x, \nu_A(x), \mu_A(x) > : x \in X \}$

(iv) $A \cap B = \{ < x, (\mu_{A\cap B})(x), (\nu_{A\cap B})(x) > : x \in X \}$ where

$(\mu_{A\cap B})(x) = \min\{ \mu_A(x), \mu_B(x) \} = \min\{ \mu_{A_i}(x), \mu_{B_i}(x) \}, \forall\ i=1,2,\ldots,k$ and

$(\nu_{A\cap B})(x) = \max\{ \nu_A(x), \nu_B(x) \} = \max\{ \nu_{A_i}(x), \nu_{B_i}(x) \}, \forall\ i=1,2,\ldots,k.$

(v) $A \cup B = \{ < x, (\mu_{A\cup B})(x), (\nu_{A\cup B})(x) > : x \in X \}$ where

$(\mu_{A\cup B})(x) = \max\{ \mu_A(x), \mu_B(x) \} = \max\{ \mu_{A_i}(x), \mu_{B_i}(x) \}, \forall\ i=1,2,\ldots,k$ and

$(\nu_{A\cup B})(x) = \min\{ \nu_A(x), \nu_B(x) \} = \min\{ \nu_{A_i}(x), \nu_{B_i}(x) \}, \forall\ i=1,2,\ldots,k.$

Here $\{ \mu_{A_i}(x), \mu_{B_i}(x) \}$ represents the corresponding i^{th} position membership values of A and B respectively. Also, $\{ \nu_{A_i}(x), \nu_{B_i}(x) \}$ represents the corresponding i^{th} position non-membership values of A and B respectively.

2.8 Definition [13]

Let A and B be any two IMFS's of groups G_1 and G_2 respectively. Then the **Cartesian product** of A and B is denoted by $A \times B$, of $G_1 \times G_2$ is defined as:

$A\times B = \{ <(p,q), \mu_{A\times B}(p,q), \nu_{A\times B}(p, q) > : (p, q)\in G_1\times G_2\}$ where

$\mu_{A\times B}(p,q) = \min\{\mu_A(p), \mu_B(q)\}$ and $\nu_{A\times B}(p, q) = \max\{\nu_A(p), \nu_B(q)\}$.

2.9 Definition [7, 8]

A mapping f from a group G_1 into a group G_2 is said to be a **homomorphism** if for all a, $b\in G_1$, $f(ab) = f(a)f(b)$.

2.10 Definition [7, 8]

A mapping f from a group G_1 into a group G_2 is said to be **anti-homomorphism** if for all a, $b\in G_1$, $f(ab) = f(b)f(a)$.

2.11 Definition [8]

An intuitionistic multi-fuzzy set (In short IMFS) $A = \{ < x , \mu_A(x), \nu_A(x) > : x\in G \}$ of a group G is said to be an **intuitionistic multi-fuzzy subgroup** of G (In short IMFSG) if it satisfies :

(i) $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$ and

(ii) $\nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}$, $\forall$ $x,y\in G$.

2.12 Remark [8]

(i) If A is an IFS of a group G, then the complement A^c is also an IFS of G.

(ii) A is an IMFSG of a group G $\Leftrightarrow$ for each i, IFS $\{< x, \mu_{A_i}(x), \nu_{A_i}(x) > : x\in G\}$ is an IFSG of group G.

2.13 Theorem [8]

If $\{ A_i: i\in I\}$ is a family of intuitionistic multi-fuzzy subgroups of a group G where $A_i = \{< x, \mu_{A_i}(x), \nu_{A_i}(x) >: x\in G\}$, then $\bigcap_i A_i$ is also intuitionistic multi-fuzzy subgroup of G.

2.14 Theorem [8]

Let A and B be any two IMFSG's of a group G. Then $A\cup B$ need not be IMFSG of G.

2.15 Theorem [8]

Let f: $G_1 \rightarrow G_2$ be anonto, homomorphism of groups G_1 and G_2. If $A=\{<x, \mu_A(x), \nu_A(x)>: x\in G_1\}$ is an intuitionistic multi-fuzzy subgroup of G_1, then $f(A) = \{< y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / y \in G_2, \text{where } y = f(x)\}$ is also an intuitionistic multi-fuzzy subgroup of G_2, if μ_A has sup property; ν_A has inf property and μ_A, ν_A are f-invariants.

2.16 Theorem [8]

Let G_1 and G_2 be any two groups. Let $f : G_1 \rightarrow G_2$ be a homomorphism of groups. If B = $\{ < y, \mu_B(y), \nu_B(y) > : y\in G_2 \}$ is an IMFSG of G_2, then $f^{-1}(B) = \{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)> : x\in G_1 \}$ is also an IMFSG of G_1.

2.17 Theorem [8]

Let G_1 and G_2 be any two groups. Let $f : G_1 \rightarrow G_2$ be an onto, anti-homomorphism. If A is an IMFSG of G_1, then f(A) is also an IMFSG of G_2 if μ_A has sup property; ν_A has inf property and μ_A, ν_A are f-invariants.

2.18 Theorem [8]

Let G_1 and G_2 be any two groups. Let $f : G_1 \rightarrow G_2$ be an anti-homomorphism. If B is an IMFSG of G_2, then $f^{-1}(B)$ is also an IMFSG of G_1.

2.19 Theorem [8]

Let A and B be any two IMFSG's of groups G_1 and G_2 respectively. Then their Cartesian product A×B is also IMFSG of $G_1 \times G_2$.

2.20 Theorem [8]

Let A be anintuitionistic multi-fuzzy set of a group G and let $\langle A \rangle = \bigcap_i \{B_i / A \subseteq B_i$ and B_i is an intuitionistic multi-fuzzy subgroup of G$\}$. Then $\langle A \rangle$ is an intuitionistic multi-fuzzy subgroup of G.

3. Properties of intuitionistic multi-fuzzy normal subgroup

In this section, we introduce the concept of intuitionistic multi-fuzzy normal subgroup (In short IMFNSG) of a group and discussed some of its related properties.

3.1 Definition

An IMFSG A = { < x , $\mu_A(x)$, $\nu_A(x)$ > : $x\in G$ } of a group G is said to be an **intuitionistic multi-fuzzy normal subgroup** (In short IMFNSG) of G if it satisfies :

(i) $\mu_A(xy) = \mu_A(yx)$ and

(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x,y\in G$.

3.2 Theorem

An IMFSG A of agroup G is said to be an IMFNSG if it satisfies for all x, $g\in G$, $\mu_A(g^{-1}xg)=\mu_A(x)$ and $\nu_A(g^{-1}xg)=\nu_A(x)$.

Proof: Let x, $g\in G$.

Then $\mu_A(g^{-1}xg)=\mu_A(g^{-1}(xg))$

$=\mu_A((xg)g^{-1})$, since A is IMFNSG of G.

$=\mu_A(x(gg^{-1}))=\mu_A(xe)=\mu_A(x)$.

Now, $\nu_A(g^{-1}xg)=\nu_A(g^{-1}(xg))$

$=\nu_A((xg)g^{-1})$, since A is IMFNSG of G.

$=\nu_A(x(gg^{-1}))=\nu_A(xe)=\nu_A(x)$. Hence the Theorem.

3.3 Theorem

If { A_i: $i\in I$} is a family of IMFNSG's of a group G, then $\bigcap_i A_i$ is also IMFNSG of G.

Proof: Let $A = \underset{i}{\cap} A_i$.

By Theorem 2.13, $\underset{i}{\cap} A_i$ is an IMFSG of G.

For any x, g∈G, $\mu_A(gxg^{-1}) = \mu_{\cap A_i}(gxg^{-1})$

$= \min_i \mu_{A_i}(gxg^{-1})$

$= \min_i \mu_{A_i}(x)$

$= \mu_{\cap A_i}(x)$

$= \mu_A(x)$

That is, $\mu_A(gxg^{-1}) = \mu_A(x)$, ∀x, g∈G.

Also, $\nu_A(gxg^{-1}) = \nu_{\cap A_i}(gxg^{-1})$

$= \max_i \nu_{A_i}(gxg^{-1})$

$= \max_i \nu_{A_i}(x)$

$= \nu_{\cap A_i}(x)$

$= \nu_A(x)$

That is, $\nu_A(gxg^{-1}) = \nu_A(x)$, ∀x, g∈G.

Hence, $A = \underset{i}{\cap} A_i$ is an IMFNSG of G.

3.4 Theorem

Union of two IMFNSG's of a group G need not be an IMFNSG of G.

Proof: Since, by Theorem 2.14, union of two IMFSG's of a group G need not be an IMFSG of G and hence the proof is clear.

3.5 Theorem

Let A be an IMFNSG of a group G. Then for all x, y$\in$G,

(i) $\mu_A(x) < \mu_A(y) \Rightarrow \mu_A(x) = \mu_A(xy) = \mu_A(yx)$ and

(ii) $\nu_A(x) > \nu_A(y) \Rightarrow \nu_A(x) = \nu_A(xy) = \nu_A(yx)$.

Proof: (i)Let A be an IMFNSG of a group G.

$$\Leftrightarrow \mu_A(xy) = \mu_A(yx) \text{ and } \nu_A(xy) = \nu_A(yx),\ \forall\ x, y\in G.........(1)$$

Suppose that $\mu_A(x) < \mu_A(y)$ for some x, y$\in$G.

Then $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$

$= \mu_A(x)$, by hypothesis.

That is, $\mu_A(xy) \geq \mu_A(x)$ ....(2)

Now, $\mu_A(x) = \mu_A(xyy^{-1})$

$\geq \min\{\mu_A(xy), \mu_A(y^{-1})\}$

$= \min\{\mu_A(xy), \mu_A(y)\}$

$= \mu_A(xy)$

Therefore, $\mu_A(x) \geq \mu_A(xy)$ ....(3)

From (2) and (3), we get $\mu_A(x) = \mu_A(xy)$ and by using (1),

$\mu_A(x) = \mu_A(xy) = \mu_A(yx)$, $\forall$ x, y$\in$G. Hence (i).

(ii)Let A be an IMFNSG of a group G.

$$\Leftrightarrow \mu_A(xy) = \mu_A(yx) \text{ and } \nu_A(xy) = \nu_A(yx),\ \forall\ x, y \in G.........(1)$$

Suppose that $\nu_A(x) > \nu_A(y)$ for some x, y∈G.

Then $\nu_A(xy) \le \max\{\nu_A(x), \nu_A(y)\}$

$= \nu_A(x)$, by hypothesis.

That is, $\nu_A(xy) \le \nu_A(x)$ ……………………………………………..………..(4)

Now, $\nu_A(x) = \nu_A(xyy^{-1})$

$\le \max\{\nu_A(xy), \nu_A(y^{-1})\}$

$= \max\{\nu_A(xy), \nu_A(y)\}$

$= \nu_A(xy)$

Therefore, $\nu_A(x) \le \nu_A(xy)$ …………………………………………………..(5)

From (4) and (5), we get $\nu_A(x) = \nu_A(xy)$ and by using (1),

$\nu_A(x) = \nu_A(xy) = \nu_A(yx)$, ∀ x, y∈G. Hence(ii).

3.6 Remark

The above Theorem 3.5 fails, if we replace in the hypothesis:

(i) $\mu_A(x) < \mu_A(y)$ by $\mu_A(x) \le \mu_A(y)$, ∀ x, y∈G.

(ii) $\nu_A(x) > \nu_A(y)$ by $\nu_A(x) \ge \nu_A(y)$, ∀ x, y∈G.

3.7 Definition

Let A be an IMFS of a group G and let $\langle A\rangle = \bigcap_i \{B_i / A \subseteq B_i$ and B_i is an IMFNSG of G$\}$.

Then ⟨A⟩ is called the IMFNSG of G generated by A. Here, note that $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$, ∀ x∈G.

International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

3.8 Theorem

Let A be an IMFS of a group G and let $\langle A\rangle = \bigcap_i \{B_i / A \subseteq B_i$ and B_i is an IMFNSG of G$\}$.

Then $\langle A\rangle$ is an IMFNSG of G.

Proof: By Theorem 2.20, $\langle A\rangle$ is an IMFSG of G.

Let $A \subseteq B_i$ and B_i be an IMFNSG of G, $\forall$ i. Also given $\langle A\rangle = \bigcap_i B_i$.

Then $\forall$ x, y$\in$G,

$$\Rightarrow \mu_{\langle A\rangle}(xy) = \mu_{\bigcap_i B_i}(xy) \quad \text{and} \quad \nu_{\langle A\rangle}(xy) = \nu_{\bigcap_i B_i}(xy)$$

$$\Rightarrow \mu_{\langle A\rangle}(xy) = \min_i \mu_{B_i}(xy) \quad \text{and} \quad \nu_{\langle A\rangle}(xy) = \max_i \nu_{B_i}(xy)$$

$$\Rightarrow \mu_{\langle A\rangle}(xy) = \min_i \mu_{B_i}(yx) \quad \text{and} \quad \nu_{\langle A\rangle}(xy) = \max_i \nu_{B_i}(yx)$$

$$\Rightarrow \mu_{\langle A\rangle}(xy) = \mu_{\bigcap_i B_i}(yx) \quad \text{and} \quad \nu_{\langle A\rangle}(xy) = \nu_{\bigcap_i B_i}(yx)$$

$$\Rightarrow \mu_{\langle A\rangle}(xy) = \mu_{\langle A\rangle}(yx) \quad \text{and} \quad \nu_{\langle A\rangle}(xy) = \nu_{\langle A\rangle}(yx)$$

Therefore, $\langle A\rangle$ is an IMFNSG of G.

3.9 Remarks

1. $\langle A\rangle$ is the IMFNSG of group G generated by A.
2. $\langle A\rangle$ is the smallest IMFNSG of group G which contains A.

4. Cartesian Product of intuitionistic multi-fuzzy normal subgroups

In this section, we introduce the concept of Cartesian product of intuitionistic multi-fuzzy normal subgroups and discuss some of its related properties.

4.1 Theorem

Let A and B be any two IMFNSG's of groups G_1 and G_2 respectively. Then their Cartesian product $A\times B$ is also an IMFNSG of $G_1\times G_2$.

Proof: By Theorem 2.19, the Cartesian product $A\times B$ is an IMFSG of $G_1\times G_2$.

Claim: $A\times B$ is an IMFNSG of $G_1\times G_2$.

Let $(p, q), (r, s) \in G_1\times G_2$.Then

$$\mu_{A\times B}((p,q)(r,s)) = \mu_{A\times B}(pr, qs)$$

$$= \min\{\mu_A(pr), \mu_B(qs)\}$$

$$= \min\{\mu_A(rp), \mu_B(sq)\}, \text{ since A \& B are IMFNSG's of } G_1 \text{ and } G_2.$$

$$= \mu_{A\times B}(rp, sq)$$

$$= \mu_{A\times B}((r,s)(p,q))$$

That is, $\mu_{A\times B}((p,q)(r,s)) = \mu_{A\times B}((r,s)(p,q))$.

$$\nu_{A\times B}((p,q)(r,s)) = \nu_{A\times B}(pr, qs)$$

$$= \max\{\nu_A(pr), \nu_B(qs)\}$$

$$= \max\{\nu_A(rp), \nu_B(sq)\}, \text{ since A \& B are IMFNSG's of } G_1 \text{ and } G_2.$$

$$= \nu_{A\times B}(rp, sq)$$

$$= \nu_{A\times B}((r,s)(p,q))$$

That is, $\nu_{A\times B}((p,q)(r,s)) = \nu_{A\times B}((r,s)(p,q))$.

Hence, $\mu_{A\times B}((p,q)(r,s)) = \mu_{A\times B}((r,s)(p,q))$ and $\nu_{A\times B}((p,q)(r,s)) = \nu_{A\times B}((r,s)(p,q))$

Hence, $A\times B$ is an IMFNSG of $G_1\times G_2$.

4.2 Remark

Let A and B be IMFS's of G_1 and G_2 respectively. If $A \times B$ is an IMFNSG of $G_1 \times G_2$, then it is not necessarily that both A and B are IMFNSG's of G_1 and G_2 respectively.

5. Properties of an intuitionistic multi-fuzzy normal subgroup of a group under homomorphism and anti-homomorphism

In this section, we discuss the properties of an intuitionistic multi-fuzzy normal subgroup of a group under homomorphism and anti-homomorphism.

5.1 Theorem

Let $f : G_1 \to G_2$ be an onto, homomorphism of groups. If $A = \{ < x , \mu_A(x), \nu_A(x) > : x \in G_1 \}$ is an IMFNSG of G_1, then $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / y \in G_2, \text{where } y = f(x) \}$ is also an IMFNSG of G_2 if μ_A has sup property; ν_A has inf property and μ_A, ν_A are f-invariants.

Proof: By Theorem 2.15, f(A) is an IMFSG of G_2.

Let A be an IMFNSG of group G_1.

Let $y_1, y_2 \in G_2$.

Since f is onto, there exist elements $x_1, x_2 \in G_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$.

Since A is an IMFNSG of G_1, $\mu_A(x_1x_2) = \mu_A(x_2x_1)$ and $\nu_A(x_1x_2) = \nu_A(x_2x_1)$.

Also, $y_2y_1 = f(x_2)f(x_1) = f(x_2x_1)$, since f is a homomorphism.

Now, $\mu_{f(A)}(y_1y_2) = \mu_{f(A)}(f(x_1)f(x_2))$

$= \mu_{f(A)}(f(x_1x_2))$, since f is a homomorphism.

$= \mu_A(x_1x_2)$,

$\geq \min\{ \mu_A(x_1), \mu_A(x_2) \}$

$= \min\{ \mu_{f(A)} f(x_1) , \mu_{f(A)} f(x_2) \}$

$= \mu_{f(A)} f(x_2x_1)$

$= \mu_{f(A)}(y_2y_1)$, since f is a homomorphism.

That is, $\mu_{f(A)}(y_1y_2) = \mu_{f(A)}(y_2y_1), \forall\, y_1, y_2 \in G_2$.

Also, $\nu_{f(A)}(y_1y_2) = \nu_{f(A)}(f(x_1)f(x_2))$

$= \nu_{f(A)}(f(x_1x_2))$, since f is a homomorphism.

$= \nu_A(x_1x_2)$

$\leq \max\{ \nu_A(x_1) , \nu_A(x_2) \}$

$= \max\{ \nu_{f(A)} f(x_1) , \nu_{f(A)} f(x_2) \}$

$= \nu_{f(A)} f(x_2x_1)$

$= \nu_{f(A)}(y_2y_1)$, since f is a homomorphism.

That is, $\nu_{f(A)}(y_1y_2) = \nu_{f(A)}(y_2y_1), \forall\, y_1, y_2 \in G_2$.

Hence, f(A) is an IMFNSG of G_2.

5.2 Theorem

Let G_1 and G_2 be any two groups. Let $f : G_1 \rightarrow G_2$ be a homomorphism of groups. If B = $\{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in G_2 \}$ is an IMFNSG of G_2, then $f^{-1}(B) = \{ \langle x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) \rangle : x \in G_1 \}$ is also an IMFNSG of G_1.

Proof: By Theorem 2.16, $f^{-1}(B)$ is an IMFSG of G_1.

Let B be an IMFNSG of G_2.

For any x, $y \in G_1$,

$\mu_{f^{-1}(B)}(xy) = \mu_B(f(xy))$

$= \mu_B(f(x)f(y))$, since f is a homomorphism.

International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$= \mu_B(f(y)f(x))$, since B is an IMFNSG of G_2.

$= \mu_B(f(yx))$, since f is a homomorphism.

Therefore, $\mu_{f^{-1}(B)}(xy) = \mu_{f^{-1}(B)}(yx)$, $\forall$ x, y$\in G_1$.

For any x, y$\in G_1$,

$\nu_{f^{-1}(B)}(xy) = \nu_B(f(xy))$

$= \nu_B(f(x)f(y))$, since f is a homomorphism.

$= \nu_B(f(y)f(x))$, since B is an IMFNSG of G_2.

$= \nu_B(f(yx))$, since f is a homomorphism.

Therefore, $\nu_{f^{-1}(B)}(xy) = \nu_{f^{-1}(B)}(yx)$, $\forall$ x, y$\in G_1$.

Hence, $f^{-1}(B)$ is an IMFNSG of G_1.

5.3 Theorem

Let G_1 and G_2 be any two groups. Let $f:G_1 \rightarrow G_2$ be an onto, anti-homomorphism. If A = $\{ < x , \mu_A(x), \nu_A(x) > : x\in G \}$ is an IMFNSG of G_1, then $f(A) = \{< x, \mu_{f(A)}(x), \nu_{f(A)}(x) > : x\in G \}$ is also an IMFNSG of G_2 if μ_A has sup property; ν_A has inf property and μ_A, ν_A are f-invariants.

Proof: By Theorem 2.17, f(A) is an IMFSG of G_2.

Let A be an IMFNSG of G_1.

For every x, y$\in G_1$, there exist f(x), f(y)$\in G_2$.

Since A is an IMFNSG of G_1, $\mu_A(xy) = \mu_A(yx)$ and $\nu_A(xy) = \nu_A(yx)$.

Now, $\mu_{f(A)}(f(x)f(y)) = \mu_{f(A)}(f(yx))$, since f is an anti-homomorphism.

$= \mu_A(yx)$

$= \mu_A(xy)$

International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$= \mu_{f(A)}(f(xy))$

$= \mu_{f(A)}(f(y)f(x))$, since f is an anti-homomorphism.

Therefore, $\mu_{f(A)}(f(x)f(y)) = \mu_{f(A)}(f(y)f(x))$.

And $\nu_{f(A)}(f(x)f(y)) = \nu_{f(A)}(f(yx))$, since f is an anti-homomorphism.

$= \nu_A(yx)$

$= \nu_A(xy)$

$= \nu_{f(A)}(f(xy))$

$= \nu_{f(A)}(f(y)f(x))$, since f is an anti-homomorphism.

Therefore, $\nu_{f(A)}(f(x)f(y)) = \nu_{f(A)}(f(y)f(x))$.

Hence, f(A) is an IMFNSG of G_2.

5.4 Theorem

Let G_1 and G_2 be any two groups. Let $f:G_1 \to G_2$ be an anti-homomorphism. If B $= \{ < y, \mu_B(y), \nu_B(y) > : y \in G_2 \}$ is an IMFNSG of G_2, then $f^{-1}(B) = \{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > : x \in G_1 \}$ is also an IMFNSG of G_1.

Proof: By Theorem 2.18, $f^{-1}(B)$ is an IMFSG of G_1.

Let B be an IMFNSG of G_2.

For any x, $y \in G_1$,

$\mu_{f^{-1}(B)}(xy) = \mu_B(f(xy))$

$= \mu_B(f(y)f(x))$, since f is an anti-homomorphism.

$= \mu_B(f(x)f(y))$, since B is an IMFNSG of G_2.

$= \mu_B(f(yx))$, since f is an anti-homomorphism.

International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$= \mu_{f^{-1}(B)}(yx)$

Therefore, $\mu_{f^{-1}(B)}(xy) = \mu_{f^{-1}(B)}(yx)$, $\forall$ x, y$\in G_1$ and

For any x, y$\in G_1$,

$\nu_{f^{-1}(B)}(xy) = \nu_B(f(xy))$

$= \nu_B(f(y)f(x))$, since f is an anti-homomorphism.

$= \nu_B(f(x)f(y))$, since B is an IMFNSG of G_2.

$= \nu_B(f(yx))$, since f is an anti-homomorphism.

$= \nu_{f^{-1}(B)}(yx)$

Therefore, $\nu_{f^{-1}(B)}(xy) = \nu_{f^{-1}(B)}(yx)$, $\forall$ x, y$\in G_1$.

Hence, $f^{-1}(B)$ is an IMFNSG of G_1.

5.5 Theorem

Let G_i (for i = 1, 2, 3, 4) be groups. Let $f:G_1\times G_2\rightarrow G_3\times G_4$ be an onto homomorphism (or anti-homomorphism) of groups. Let A and B be any two IMFNSG's of G_1 and G_2 respectively. Let $f_1:G_1\rightarrow G_3$ and $f_2:G_2\rightarrow G_4$ be onto homomorphism (or anti-homomorphism) of groups. If A×B is an IMFNSG of $G_1\times G_2$, then f(A×B) is also an IMFNSG of $G_3\times G_4$ if A×B have sup property and also A×B is f-invariant.

Proof: It is clear.

5.6 Theorem

Let G_i (for i = 1, 2, 3, 4) be groups. Let $f:G_1\times G_2\rightarrow G_3\times G_4$ be a homomorphism (or anti-homomorphism) of groups. Let C and D be any two IMFNSG's of G_3 and G_4 respectively. Let $f_1: G_1\rightarrow G_3$ and $f_2: G_2\rightarrow G_4$ be a homomorphism (or anti-homomorphism) of groups. If C×D is an IMFNSG of $G_3\times G_4$, then $f^{-1}(C\times D)$ is also an IMFNSG of $G_1\times G_2$.

Proof: It is clear.

International Journal of Engineering, Science and Mathematics
Vol. 10 Issue 09, September 2021,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

5.7 Theorem

Let G_i (for i = 1, 2, 3, 4) be groups. Let A and B be any two IMFNSG's of G_1 and G_2 respectively. Let $f_1:G_1\rightarrow G_3$ and $f_2:G_2\rightarrow G_4$ be onto homomorphism (or anti-homomorphism) of groups. Let $f:G_1\times G_2\rightarrow G_3\times G_4$ be an onto homomorphism (or anti-homomorphism) of groups such that $f((u, v)) = (f_1(u), f_2(v))$. If A×B is an IMFNSG of $G_1\times G_2$, then $f(A\times B) = f_1(A)\times f_2(B)$ if A×B have sup property and also A×B is f-invariant.

Proof: Let A×B be an IMFNSG of $G_1\times G_2$.

Let $(u, v)\in G_1\times G_2$. Then $u\in G_1$ and $v\in G_2$. It implies that $f_1(u)\in G_3$ and $f_2(v)\in G_4$. Therefore, $(u, v)\in G_1\times G_2 \Rightarrow f((u, v)) = (f_1(u), f_2(v))\in G_3\times G_4$. Then

$$\begin{aligned}
\mu_{f(A\times B)}(f_1(u), f_2(v)) &= \mu_{f(A\times B)}(f(u, v)) \\
&= \mu_{A\times B}(u, v) \\
&= \min\{\mu_A(u), \mu_B(v)\} \\
&= \min\{\mu_{f_1(A)}(f_1(u)), \mu_{f_2(B)}(f_2(v))\} \\
&= \mu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))
\end{aligned}$$

Therefore, $\mu_{f(A\times B)}(f_1(u), f_2(v)) = \mu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))$, for all $(f_1(u), f_2(v))\in G_3\times G_4$.

$$\begin{aligned}
\nu_{f(A\times B)}(f_1(u), f_2(v)) &= \nu_{f(A\times B)}(f(u, v)) \\
&= \nu_{A\times B}(u, v) \\
&= \max\{\nu_A(u), \nu_B(v)\} \\
&= \max\{\nu_{f_1(A)}(f_1(u)), \nu_{f_2(B)}(f_2(v))\} \\
&= \nu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))
\end{aligned}$$

Therefore, $\nu_{f(A\times B)}(f_1(u), f_2(v)) = \nu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))$, for all $(f_1(u), f_2(v))\in G_3\times G_4$.

Hence, $f(A\times B) = f_1(A)\times f_2(B)$.

5.8 Theorem

Let G_i(for i = 1, 2, 3, 4) be groups. Let C and D be any two IMFNSG's of G_3 and G_4 respectively. Let $f_1: G_1 \to G_3$ and $f_2: G_2 \to G_4$ be homomorphism (or anti-homomorphism) of groups. Let $f: G_1\times G_2 \to G_3\times G_4$ be a homomorphism (or anti-homomorphism) such that $f((u, v)) = (f_1(u), f_2(v))$. If $C\times D$ is an IMFNSG of $G_3\times G_4$, then $f^{-1}(C\times D) = f_1^{-1}(C)\times f_2^{-1}(D)$.

Proof: Let $C\times D$ be an IMFNSG of $G_3\times G_4$.

Let $(u,v)\in G_1\times G_2$. Then $u\in G_1$ and $v\in G_2$. It implies that $f_1(u)\in G_3$ and $f_2(v)\in G_4$. Therefore, $(u, v)\in G_1\times G_2$.

$\Rightarrow f((u, v)) = (f_1(u), f_2(v))\in G_3\times G_4$, since f is homomorphism.

Then $\mu_{f^{-1}(C\times D)}(u,v) = \mu_{C\times D}\, f((u,v))$

$= \mu_{C\times D}(f_1(u), f_2(v))$

$= \min\{\mu_C(f_1(u)), \mu_D(f_2(v))\}$

$= \min\{\mu_{f_1^{-1}(C)}(u), \mu_{f_2^{-1}(D)}(v)\}$

$= \mu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u,v)$

Therefore, $\mu_{f^{-1}(C\times D)}(u,v) = \mu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u,v)$, for all $(u,v)\in G_1\times G_2$.

And $\nu_{f^{-1}(C\times D)}(u,v) = \nu_{C\times D}\, f((u,v))$

$= \nu_{C\times D}(f_1(u), f_2(v))$

$= \max\{\nu_C(f_1(u)), \nu_D(f_2(v))\}$

$= \max\{\nu_{f_1^{-1}(C)}(u), \nu_{f_2^{-1}(D)}(v)\}$

$= \nu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u,v)$

Therefore, $\nu_{f^{-1}(C\times D)}(u,v) = \nu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u,v)$, for all $(u,v)\in G_1\times G_2$.

Hence, $f^{-1}(C\times D) = f_1^{-1}(C) \times f_2^{-1}(D)$.

6. CONCLUSION

The intuitionistic multi-fuzzy sets are very important role for the development of the theory of intuitionistic multi-fuzzy subgroups. In this paper an attempt has been made to study some new algebraic structures of intuitionistic multi-fuzzy normal subgroups and their properties were discussed.

REFERENCES

[1] AtanassovK.T., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), No.1,87-96.

[2] Basnet D.K. and Sarma N.K., A note on Intuitionistic Fuzzy Equivalence Relation, International Mathematical Forum, 5, 2010, No.67, 3301-3307.

[3] Biswas R., Vague Groups, International Journal of Computational Cognition, Vol.4, No.2, June 2006.

[4] Das P.S., Fuzzy groups and level subgroups, Journal of Mathematical Analysis and Applications, 84 (1981), 264-269.

[5] KulHur and Su Youn Jang, The lattice of Intuitionistic fuzzy congruences, International Mathematical Forum, 1, 2006, No.5, 211-236.

[6] Mukharjee N.P. and Bhattacharya P., Fuzzy normal subgroups and fuzzy cosets, Information Sciences, 34 (1984), 225-239.

[7] Muthuraj R. and Balamurugan S., Multi-fuzzy group and its Level subgroups, Gen. Math. Notes, Vol. 17, No.1, July, 2013, pp. 74-81.

[8] Muthuraj.R and Balamurugan.S., "A Study on Intuitionistic Multi-Fuzzy Subgroups", Intl. Jour. of Applications of Fuzzy Sets and Artificial Intelligence (IJAFSAI), Vol. 4 (July 2014), pp.153-172.

[9] Rosenfeld A., Fuzzy groups, Journal of Mathematical Analysis and Applications, 35 (1971), 512-517.

[10] Sabu S. and Ramakrishnan T.V., Multi-fuzzy sets, International Mathematical Forum,50 (2010), 2471-2476.

[11] Sabu S. and Ramakrishnan T.V., Multi-fuzzy subgroups, Int. J. Contemp. Math.

Sciences,Vol.6, 8 (2011), 365-372.

[12] Sharma P.K., Intuitionistic Fuzzy Groups, ifrsa International Journal of Data warehousing and Mining, vol.1, 2011, iss.1, 86-94.

[13] Shinoj T.K. and Sunil Jacob John, Intuitionistic Fuzzy Multisets, Intl. Journal of Engineering Science and Innovative Technology, Vol.2, 2013, Issue 6, 1-24.

[14] Zadeh L.A., Fuzzy Sets, Information and Control 8, (1965), 338-353.
